

Data-driven system identification of the social network dynamics in online postings of an extremist group

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Abstract—*Terrorism research has begun to focus on the issue of radicalization, or the acceptance of ideological belief systems that lead toward violence. There has been particular attention paid to the role of the Internet in the exposure to and promotion of radical ideas. There is, however, minimal work that attempts to model the ways that messages are spread or how individual participation in radical on-line communities operates. In this paper, we present a stochastic linear system to represent the evolution of contribution to a sample of 126 threads in an on-line forum where individuals discuss radical belief systems. To estimate or predict the time-varying contributions of agents for given online-forum data, each agent's contribution has been modeled as a state variable. We then use the expectation-maximization (EM) algorithm to identify the model parameters including the adjacency matrix of the graph constructed among participating agents along with measurement and system uncertainty levels in online-postings. Our approach reveals the identified dynamical influences among agents in the time-varying shaping of the contribution in a data-driven fashion. We use the real-world data from online-postings to demonstrate the usefulness of our approach, and its application toward on-line radicalization.*

Keywords— cyberterrorism; radicalization; Kalman filter; EM algorithm; forecasting; maximum likelihood estimation.

I. INTRODUCTION

Terrorism research has dramatically expanded in the last two decades, with 90% of the publications associated with this field being generated after the 9/11 attacks in the United States [1]. There has been particular growth in empirical examinations of the foreground and situational dynamics that may lead individuals to engage in ideologically motivated acts of violence, which is also referred to as a radicalization process [2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12]. Much of this research is driven by continued growth in both domestic and international terror groups and movements, as well as continued high profile terror attacks [2, 11, 12, 13].

There is also substantial concern over the ways that terror and extremist groups use social media and the Internet as a vehicle for recruitment [11]. There have been several notable cases of individuals in the US being contacted by members of

al Qaeda and ISIS living abroad to recruit them into the movement [2, 15]. The threat of domestic radicalization occurring without any physical contact with foreign extremist movements is relatively unparalleled, and calls to question how the Internet may engender acceptance of radical belief systems without physical interaction or integration into larger actor networks [2, 11].

The Internet creates a source for ideological information sharing 24 hours a day, from any part of the globe, facilitating equal access to radical messages and networks where individuals may gain entrance to extremist groups [16, 17, 18, 19]. Social media sites like Facebook and Twitter enable individuals to identify others who self-identify as members of radical movements to freely discuss their views with little fear of reprisal [11, 20]. In addition, individuals use the Internet to identify others in geographic proximity, or arrange meetings and events in the real world [11]. In much the same way, digital media allow individuals to create videos and magazines that can be posted and shared in various social media outlets like YouTube [21]. Finally, posting videos and news stories through social media provide another mechanism to publicly refute claims from the media and government and to portray their own group in a positive light [21, 22].

As a result, the Internet is now a critical resource that terror groups use to maintain their presence and growth over time [11, 23, 24]. This is true for all forms of extremism and terror, whether white nationalists and others associated with the Far Right, or jihadist movements globally [21, 25]. The Internet's on-demand nature means that individual exposure to radical ideas and content is limited only by their willingness to spend time on-line.

One key avenue to explore radicalization may be through web forums, which have been a long-standing form of asynchronous on-line communication used by deviant [26, 27, 28] and non-deviant groups alike [29, 30, 31]. Forums act as online discussion groups where individuals present issues or discuss problems [26, 32, 33, 34]. They are composed of "threads" which begin when a registered user creates a post within a forum, asking a question or making a statement.

Other people respond to the remarks with posts of their own that are connected together to create threads. Thus, threads are composed of posts that center on a specific topic under a forum's general heading. Since posters respond to the ideas of others, the exchanges present in the strings of a forum demonstrate relationships between individuals [32, 33, 35]. These connections can serve as a key data source to understand the process of social engagement within any subculture [32, 33, 35]. In turn, posting behavior may demonstrate the ways that individuals become enculturated into an extremist group and relate to others online.

The domestic far-right in the US has long used the Internet and forums in particular to communicate with followers, recruit new members, and propagandize. In the early 1990s, public officials, watch-groups, and scholars noted that technologies were critical to the growth of the far-right militia movement [36, 37, 38, 39, 40]. While the militia movement declined in the late 1990s, it has recently surged, partially due to online discussion forums and email distribution lists [41]. Similarly, the racist far-right online site Stormfront.org was established in the 1990s [42, 43] and has attracted a massive user population and financial support from posters who directly fund forum operations and hosting [44]. In fact, Stormfront.org has emerged as a model for many groups that have resulted in "virtual communities of hate" [45, 46].

Forums provide individuals with a variety of information, such as justifications for the use of violence, and calls for attacks against general categories of enemies (e.g., the New World Order, the hedonistic West, the Zionist Occupied Government, the police, the Jews, etc.) [42]. The importance of online identities of hate groups was noted by [47] who found that the vast majority of groups involved in ideologically-motivated violence had an Internet presence.

The more time individuals spend in websites and social media may increase the quantity of messaging individuals and also enculturate them into social networks that espouse violence or radical ideas. Time spent in forums can also provide individuals with information on the argot and outward symbols of movement membership. Members of the white supremacist community demonstrate their identity through the use of linguistic indicators such as, 88 (standing for HH, or Heil Hitler), or phrases such as the 14 Words of David Lane [11, 48]. The use of text-based identifiers is key since the lack of physical cues in on-line spaces require individuals to demonstrate their group affiliation in some fashion [26].

There have, however, been few investigations of the network structures and posts made within forums used by members of extremist groups. One of the few studies of this nature assessed the content of communications in jihadist websites [49]. Evidence suggests that most forum discussions do not last long, involve a limited number of participants, and the posted entries were short in length. [49] also examined the types of posts, such as whether religious sources were referenced or if there was encouragement to commit jihad, instructions regarding military training, or social interactions.

This analysis, however, gave no information on the relationship between online posts and ideologically motivated violence committed by supporters of these forums and organizations.

As a result, the publicly accessible online activities of members of extremist movements may provide signals of the radicalization process in action. Posts in web forums provide direct, overt information on the relationships between actors in larger networks [26, 50]. Combining network analyses with the content of posts made by individuals will provide direct information on the role that posts of ideological rhetoric and information play in larger networks of extremist communities. Those who constantly post in forums may be more pivotal actors within a network, serving as hubs to link individuals and facilitate the flow of information. Such information would be invaluable to identify the flow of radical messages from group to group, and understand how posting habits and participation in multiple forms of CMC shape that person's role in the prospective radicalization of others. This data will not, however, speak to connections that may emerge between participants through private messages made via email or other mechanisms [32, 33]. Thus, this analysis can only speak to the public process of radicalization, which occurs in tandem with direct messaging between individuals in and outside of extremist groups [21, 23, 24, 48, 49].

Few researchers have considered this issue, let alone examined the network structures that underlie extremist movements online. [9] and [2] studied the radicalization process of al Qaeda-influenced terrorists and found that individuals became radicalized in clusters that were created in part through peer associations. Few have considered how network structures of extremist groups function to connect members together and prospectively radicalize individuals to violence. It is unclear if members who frequently post content are central actors in larger networks of extremists, or if they are simply producing a greater degree of noise relative to a smaller proportion of individuals who are tied to multiple groups yet are infrequently online. Further, there is minimal knowledge of the ways that far-right extremist groups with members living in close physical proximity are linked to other groups through the web. Such information is invaluable to identify ways to disrupt networks that provide messages that lead to radicalization, based on the presence of redundant relationships, resiliency of message delivery, and network density [51].

At the same time, close to 99% of persons exposed to radical messages never actually engage in violence themselves [52]. Thus, it is critical to understand how individuals are socialized into radical or extremist movements, and how individual participation in on-line movements changes over time. This study considered these questions using a set of threads collected from a nationally-known forum used by members of the Far Right to connect and discuss various issues.

To understand the social dynamics in such online-postings, we present a stochastic linear system to represent the evolution of the contribution in online-postings. To estimate or predict the contributions of agents for given web-forum data, each agent's contribution has been modeled as a state variable. We then use the expectation-maximization (EM) algorithm to estimate the model parameters including the adjacency matrix of the graph-constructed among participating agents along with measurement and system noise processes in online-postings. Our approach reveals the estimated dynamical influences among agents in the time-varying shaping of the contribution in a data-driven fashion. We use the real-world data from the major Far Right web forum to demonstrate the usefulness of our approach.

II. BUILDING A GRAPH THEORETICAL MODEL FROM FORUM POST DATA

The data set was collected from a single sub-forum of a major Far Right web forum. All 126 threads made in that forum, consisting of posts from 46 agents over a period of about 1500 days, were downloaded by the researchers and saved as complete web pages in order to maintain the posts in their original state. A thread is composed of posts, whereby an individual creates an initial message that is made available for others to see within the forum. Responses to that initial post create a thread, which resembles other forms of naturally occurring communications, particularly group discussions [33, 34]. Since anyone who is a member of the forum can read a post made in a thread at any given time, it creates the opportunity to understand the influence of asynchronous communication on the transfer of information, attitudes, and beliefs on others.

The posts from each of these threads were used to construct a network that represents relationships among agents and to assess the level of engagement of each agent in that sub forum and its evolution over time. Data are organized in *threads* $T = \{T^1, \dots, T^N\}$. Each thread is a collection of data structures containing information about each posting in the thread, including the author's (agent) identifying handle, time of posting, and content (content is not used in this work). In particular, let

$$H^\alpha = \{h_k^\alpha\} \quad \text{and} \quad T^\alpha = \{\tau_k^\alpha\}, \quad \alpha=1, \dots, N$$

for threads $\alpha=1, \dots, N$ where

- h_k^α identifies the unique handle identifying the author of the k -th posting (note that, as the same individual may post more than once in the same thread, H^α often contains repeated entries) and
- τ_k^α stores the time of posting, in number of days from a reference datum.

We use the author lists H^α to represent the data in a graph-theoretical framework. To this end, from each thread α and author list, we build a subgraph $G^\alpha = (V^\alpha, E^\alpha)$ with vertices V^α and edges E^α by letting

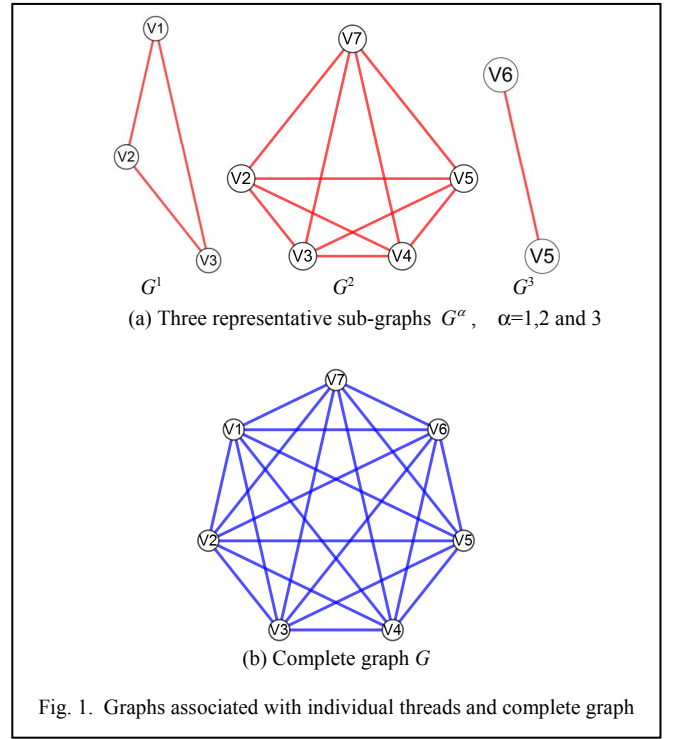


Fig. 1. Graphs associated with individual threads and complete graph

- Vertices V^α in G^α correspond to agents in H^α (removing duplicates), i.e., $V^\alpha = \{v : v \in H^\alpha\}$.
- G^α be a *complete subgraph*, i.e., $(i, j) \in E^\alpha$ if $i \in V^\alpha$ and $j \in V^\alpha$

As agents often participate in several threads, sub-graphs often share vertices and edges. The complete data set is represented by the graph $G = (V, E)$ where

$$V = \bigcup_{\alpha=1}^N V^\alpha \quad \text{and} \quad E = \bigcup_{\alpha=1}^N E^\alpha$$

To illustrate, let the data be a collection of 16 postings from seven agents with handles $\{v1, v2, v3, v4, v5, v6, v7\}$ in three threads: $H^1 = \{v1, v2, v1, v3, v1, v2\}$, $H^2 = \{v2, v3, v7, v2, v5, v2, v3\}$ and $H^3 = \{v5, v6, v5\}$. For these data

$$V^1 = \{v1, v2, v3\}, \quad V^2 = \{v2, v3, v4, v5, v7\} \quad \text{and} \quad V^3 = \{v5, v6\}.$$

The sub-graphs G^1, G^2 and G^3 associated with these data are shown in Fig. 1(a), along with the complete graph G (Fig. 1(b)).

In building the graphs G^α associated with the various threads we have made a number of important assumptions. In particular,

- Agents are connected if and only if they have contributed to the same thread.
- Agents will influence each other's engagement *directly* only if their vertices are connected in the graph.
- Graphs are undirected. This means that if $(a, b) \in E$, a is as likely to influence b 's engagement as b is likely to influence a .

- Even though agents contribute their postings at different times, we assume that connections between agents exist for all times, i.e., edges in E are not time-dependent. Note, however, that updates from EM algorithm do account for the time of participation of each agent and, implicitly, for the persistence of the connection between agents, at least in some time-averaged sense.

Later, we will explore the implications of removing some of these assumptions and ways to address them.

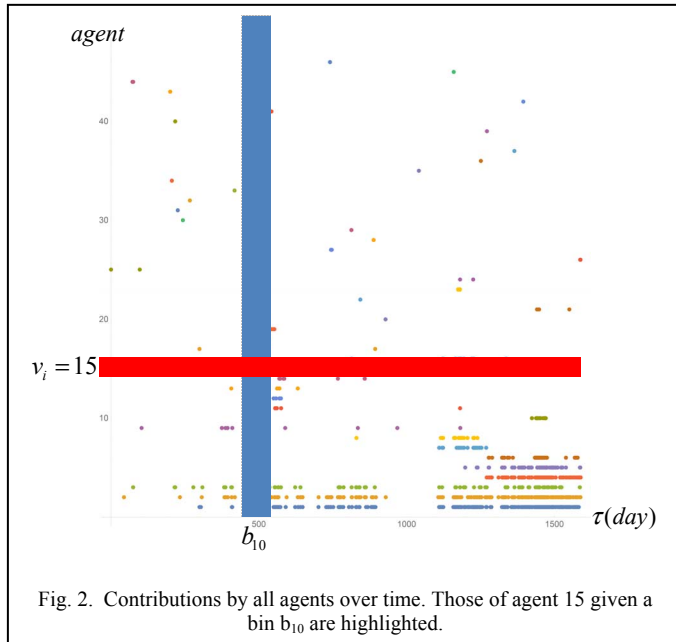
We use the information on the time of posting τ_k^α , along with the number of postings, to assess the degree of engagement of each agent in the conversation. This is described next.

III. ESTIMATING EVOLUTION OF ENGAGEMENT FROM THE THREADS

As mentioned above, network connections among agents are not likely to be formed all at once, as the forum's activity may span long time periods, possibly years, and agents may enter a thread for the first time at any time within this period. Nevertheless, we establish the network of connections among agents from the data, independently from the time at which the individual enters the thread. Time and number of contributions are used to estimate the time-evolution of engagement of each agent, as defined below.

Let the first posting in the data occur at the reference time $\tau=0$ and the last one at $\tau_{\max} = \max_{\alpha,k} \tau_k^\alpha$. For simplicity, we assume all time units are in days. We partition the total time $[0, \tau_{\max}]$ into n bins b_t of fixed size, indexed by $t = 1, \dots, n$, and count the number of postings by each agent within each bin. For instance, we let

$$b_t = [\tau_t, \tau_t + \Delta] \quad \text{and} \quad \tau_{t+1} = \tau_t + d \quad \text{with} \quad d \leq \Delta$$



Bin size Δ and increment d (both measured in days) can be selected to allow overlap (for data smoothing). With this construction, it is straight forward to extract from the data the quantity $c(i, t)$, measuring the number of postings by agent i within bin b_t .

Figure 2 shows contributions corresponding to data collected from 126 threads with contributions from 46 agents over a period of about 1500 days. The time of each posting τ is shown along the horizontal axis (measured from the first entry). In the figure, a contribution from agent i is entered as a “x” displayed a distance i from the horizontal axis, placed at the appropriate time τ . Letting $b_{10} = [450, 500]$ the plot shows that $c(15, 10) = 4$.

This model demonstrates that participation in the forum's threads is not evenly distributed. A small proportion of users are frequent posters, compared to the larger body of individuals making occasional posts in keeping with patterns of participation from research on deviant on-line communities generally [26, 34, 35]. This suggests that individuals who are frequent posters may have a greater influence on the tenor of discussions and potentially exert control over the radical nature of conversation. For terrorism research, there is value in identifying how individual participation changes over time, and if those who post more or less frequently have any change in their attitudes or beliefs.

We associate higher values of c with higher levels of engagement and expect engagement to vary from agent to agent and, for a given agent, to vary with time. We discuss how we model the dynamic evolution of agent engagement in the text section.

IV. KALMAN FILTER ESTIMATION FORMULATION FOR AGENT ENGAGEMENT

In this section, we show how to model the evolution of the state as a stochastic dynamical system that is subject to measurement and process noises. The objective is two-fold. First to estimate the state x from the observations we formulate our model as a discrete-time stochastic dynamical system. Next, we perform system identification via the expectation-maximization (EM) algorithm [54] in order to estimate the social-network influences among agents as in the formulated stochastic system.

At time t , let x_t be a $p \times 1$ vector representing an “idealized” measure of engagement of agents contributing to threads T , described in the previous section. p is the total number of agents in the data and $t=1, 2, \dots, n$ represents the sampling time associated with bins $b_1, b_2, \dots, b_n$. We assume that scaling is possible so that $0 \leq x_{t,i} \leq 1$, larger values representing more engagement.

The state vector x_t is not observed directly. Instead, we assume it appears in a random regression model of the form

$$y_t = Mx_t + z_t, \quad t=1, 2, \dots, n \quad (1)$$

x_0 is a random process with mean μ and covariance Σ . In (1), y_t is the measurement at time t , in our case extracted from the

collected data $c(i, t)$. Here y_t itself measures engagement, and M is the $p \times p$ identity matrix. The actual measurement $c(v, t)$ is as described in the previous section, namely, $c(v, t)$ represents the number of postings by agent v in measurement interval b_t . We construct y_t from c by assigning an evaluation of “full engagement at time t ” (i.e., $y=1$) whenever an individual contributes δ or more postings within interval b_t . Thus, we define for agent i

$$y_{t,i} = \min(c(i, t) / \delta, 1)$$

$i=1, 2, \dots, p$ and $t=1, 2, \dots, n$. Measurements are subject to measurement error, v_t . We assume z_t is a $p \times 1$ vector of random, uncorrelated errors with covariance matrix R and zero mean.

Transition from x_t to x_{t+1} is modeled as a first order process along with a system noise process,

$$x_t = \Phi x_{t-1} + w_t, \quad (2)$$

Here Φ is a $p \times p$ transition matrix. The model in (2) is subjected to random, uncorrelated normal distribution $p \times 1$ vector w_t with covariance matrix Q and zero mean.

Matrix Φ reflects the time-averaged interactions between agents that best fit the data set over the sampling period. An entry $\Phi(i, j)$ gives a measure the strength of the connection between agents i and j . Initially, Φ will be obtained from a linear model associated with the network graph G built from the thread data. This construction is standard, and it follows e.g., the work in [53]. Specifically, in a (deterministic) discrete time system associated with graph G , the protocol dictates that

$$x_{t+1} = C_\epsilon x_t \quad (3)$$

where

$$C_\epsilon = I - \epsilon L \quad (4)$$

I is the $p \times p$ identity matrix and ϵ is a scalar parameter, adjusted so that $\min_i |\lambda_i| < 1$ where λ_i is an eigenvalue of C_ϵ . The reader may note that this construction is typical in consensus problems with multi-agent systems [53]. In that case, $L = D - A$ is the $p \times p$ Laplacian matrix associated with graph G ; $A = [a_{ij}]$ is the graph adjacency matrix with $a_{ij} = 1$ if $(i, j) \in E$, otherwise $a_{ij} = 0$; and $[D] = \text{diag}(d_{ii})$ with $d_{ii} = \sum_j a_{ij}$. Setting the step size ϵ so that $\epsilon < 1 / \max(d_{ii})$ insures stability (see [53]).

The transition matrix Φ in (2) can be built from C_ϵ in (4), e.g., as

$$\Phi = C_\epsilon^k \text{ for } k \geq 1 \quad (5)$$

For the given measurements $y_1, \dots, y_s$ the conditional mean $x_t^s := E(x_t | y_1, \dots, y_s)$, and the conditional covariance matrices

$P_t^s := \text{cov}(x_t | y_1, \dots, y_s)$ and $P_{t,t-1}^s := \text{cov}(x_t, x_{t-1} | y_1, \dots, y_s)$ associated with this process can be obtained from the standard Kalman filter equations (see Appendix).

Recall that the objective in this paper is to identify the system in order to smooth the state for given the whole data set, i.e., to find $x_t^n := E(x_t | y_1, \dots, y_n)$, the minimum mean square error smoothed estimator of x_t based on the data [54].

V. SYSTEM IDENTIFICATION VIA EM ALGORITHM FOR TIME SERIES SMOOTHING AND FORECASTING OF AGENT ENGAGEMENT

As discussed before, construction of the graph G relies on several assumptions which affect the accuracy of our estimation of the state vector x_t . Rather than relying only on the transition matrix Φ from G and a consensus model (the initial guess), a sequence of operators $\Phi(r)$, $r=1, 2, \dots$ will be constructed based on an EM algorithm by using the available data. This is discussed next. The approach follows the strategy presented in [54].

The process is started with an initial guess x_0 , a normal random vector with mean μ and covariance matrix Σ . We start with $p \times p$ diagonal covariance matrices $R = \sigma_R^2 I$, $Q = \sigma_Q^2 I$, and $\Sigma = \sigma_\Sigma^2 I$. The EM algorithm updates covariance matrices Q , R , and Σ until iterations stabilize, according to the strategy described in [54], using formulas summarized in the Appendix.

Application of the EM algorithm seeks to smooth the state x based on the available data through an iterative update of the transition operator Φ . Thus, while the initial transition matrix is built from a consensus model applied to graph G , this system identification process allows us to identify the model based on the data and account for the uncertainty levels associated with the social dynamics in online-postings. And while in this model the transition matrix is not time dependent, updates from EM algorithm do account for the time of participation of each agent and, implicitly, for the persistence of the connection between agents, at least in some time-averaged sense.

VI. RESULTS

This example illustrates the modeling strategy using data collected over a five-year period. There are a total of $p=46$ participants in the subforum, identified by distinct handles, contributing to $N=126$ threads. Graphs associated with each thread were built as described in the previous section. A sample of 12 popular threads, as well as the complete graph, are shown in Fig. 3. Data regarding contributions over time displayed in Fig. 2 come from these threads. The data collected was organized in $n=40$ overlapping bins b_t of length $\Delta=50$ and $d=25$, starting from $\tau=500$, from where y_t was built. $\sigma_R = 0.2$, $\sigma_Q = 0.2$, $\sigma_\Sigma = 0.2$ were used for the initial covariance matrices $R = \sigma_R^2 I$, $Q = \sigma_Q^2 I$, and $\Sigma = \sigma_\Sigma^2 I$,

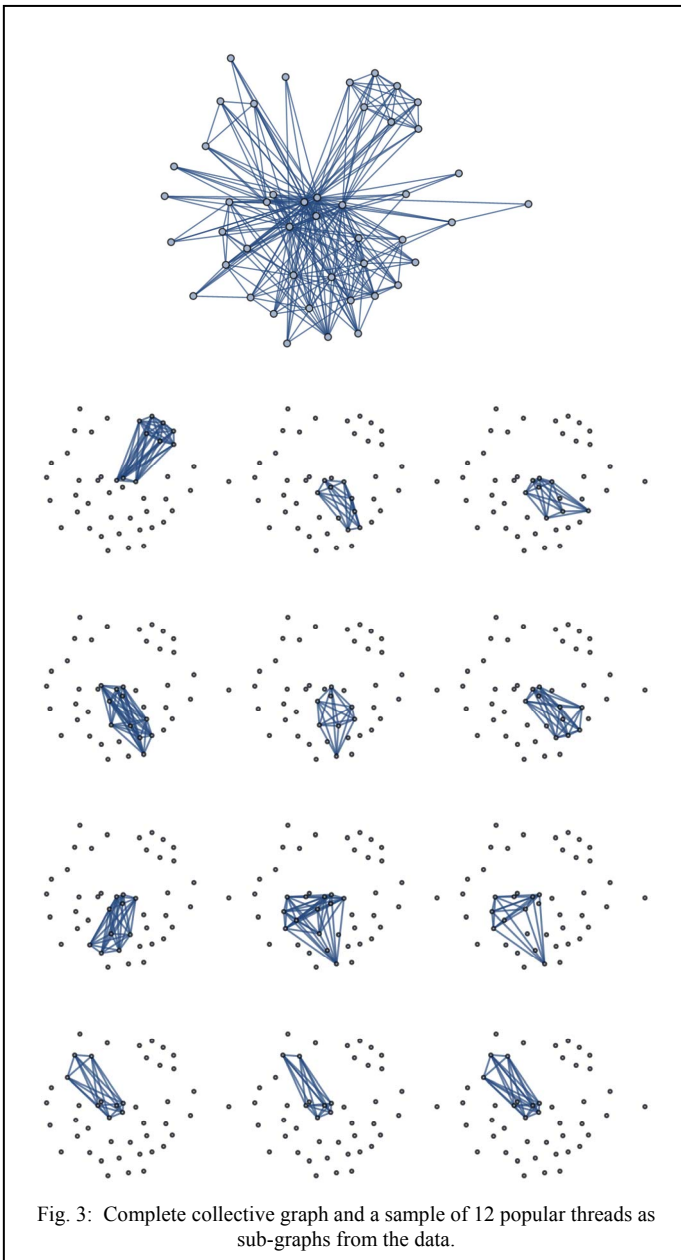


Fig. 3: Complete collective graph and a sample of 12 popular threads as sub-graphs from the data.

respectively. The EM algorithm produced the finalized transition matrix Φ and the smoothed states (orange dots) are plotted along with observations (blue dots) as shown in Fig. 4.

VII. DISCUSSION

As can be seen in Fig. 4, the identified transition matrix Φ allows us to smooth the state x successfully. By comparing both observations and the smoothed states, we conclude that they match well and so our scheme provides the reasonably identified system in a data driven way.

Sub-graphs: As shown in Figs. 3 and 4, participation in threads is consistent among a small number of users who serve to connect otherwise individual isolates who post infrequently. This finding again supports the notion that those frequent

posters may have greater influence on the discussions within the forum generally.

The identified transition matrix: Our approach reveals the influence among agents in the estimated adjacency-like matrix in a data-driven way. Note that we start the EM algorithm with a simple consensus-type model initially, i.e., $a_{ij}=1>0$ assuming that all connected agents are influencing each other with equal magnitude to promote to become similar. Without uncertainties and some mild conditions, the evolution (i.e., the consensus algorithm) will make the state entries to converge to the averaged value when the graph is connected [54]. However, the identified transition matrix show that the estimated a_{ij} and the associated adjacency matrix contains both positive and negative values, which contribute to promote similarity or dissimilarity simultaneously, among agents i and j , respectively. Positive and negative edge modeling has attracted many researchers in modeling social networks [55, 56]. We may view our findings as positive and negative edge modeling on online social networks in a dynamic system setting. This identified static adjacency matrix shows how a particular, e.g., agent i influences agent j over the evolution on average in a data-driven way.

Prediction of the future: The system identified here can be also used to compute the prediction of the states in future time based on the data. It will be of great importance to gauge the prediction capability using a larger set of longitudinal data, which can be divided by two segments to predict the later state evolution in the second segment after identification based on the first segment. Furthermore, the computed covariance matrices can allow us to compute the confidence intervals for such predicted state variables. With the positive and negative contributions between each agent, we found that the transition matrix is unstable, i.e., there exists an eigenvalue outside of the unit circle. In this case, the confidence intervals will increase as the prediction time increases.

Further refinement on model parameterization: We may constrain any model parameters according to scientific reasons in the maximization process in the EM algorithm. In contrast to identify a time-averaged transition matrix over the whole longitudinal data, we may seek for a time-varying transition matrix. In fact, by incorporating the sub-graphs in Fig. 4 in timely manner, we may find further time-varying evidence among the social network interactions. To synchronize sub-graphs as shown in Fig. 4, with our transition matrix, we may parameterize it as a time-varying one, which will increase the number of unknown parameters proportionally to that of integrations. The number of model parameters can be reduced in this case, for example, if agent i does not have chance to read the post by agent j , i.e., agent j does not have a chance to influence agent i in this dynamic process at time t , we can constrain $a_{ij}(t)=0$.

VIII. CONCLUSIONS

We presented a stochastic linear system to represent the evolution of the contribution in online-postings. The contribution or engagement of an agent was modeled as a state variable in the online-forum. The EM algorithm identified the model parameters including the adjacency matrix of the graph constructed among participating agents along with measurement and system noise processes in online-postings. Our approach revealed the identified dynamical influences among agents in the time-varying shaping of the contribution in a data-driven fashion. We used the real-world data from online-postings to demonstrate the usefulness of our approach. The smoothed state variables match well with the observations. We also discussed how to predict the future states and to refine the model in order to synchronize and take into account sub-graphs.

Further research is needed in order to further address the connection between post frequency, network position, and influence on radicalization. Specifically, the smoothed state analyses suggest participation in threads is constant for a very small number of individuals. Instead, there appears to be some factor affecting the frequency of participation for several users which may peak at a specific point in time. For those individuals who appear to be on an upward participation trajectory, it is pivotal that we understand if this will be sustained or will lead to a sudden removal from the network for some reason.

Further study is also needed to understand how the attitudes and beliefs of the individuals change during this period of increased participation toward more radical points of view. Linguistic and semantic analyses of posts and transitions in the extremist views presented are demanded in order to improve our understanding of the process of radicalization in on-line communications mediums.

There is also a need to assess changes in the differences in posting frequencies and behaviors surrounding key events in the real world. For instance, if a major terror plot either succeeds or is foiled and receives major news coverage, it is plausible this would directly affect the quantity and quality of posts made by participants in various forums. This analysis did not control for real world events and their prospective influence on posting behaviors, though such analyses would benefit our knowledge of reciprocal relationship between virtual and real events [35].

In addition, this study could not account for the influence of private messages made between participants either within this forum or in external methods of communication, including email and other forms of CMC. These hidden communications may prove essential in explicating the role of individuals acting as hubs for radicalization messages. Such data may be difficult to identify and requires innovative collection strategies in order to be obtained [32, 33]. Open source databases, such as those used to address successful, failed, and foiled extremist plots may provide a resource to

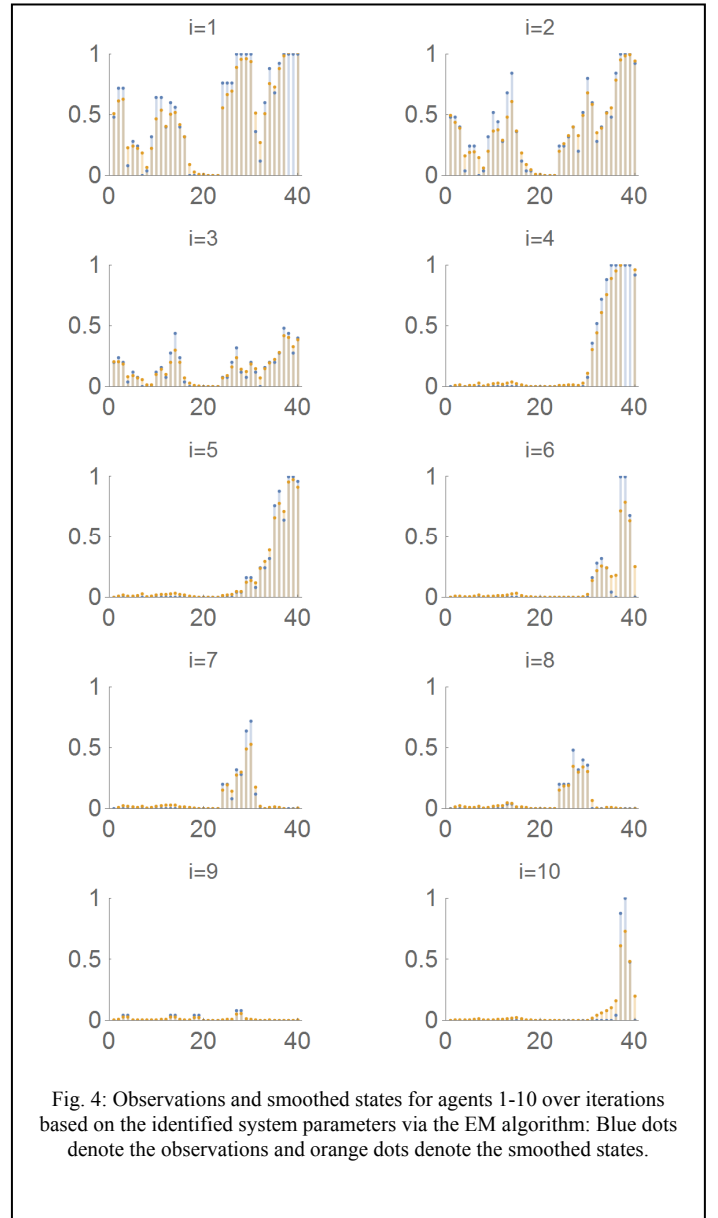


Fig. 4: Observations and smoothed states for agents 1-10 over iterations based on the identified system parameters via the EM algorithm: Blue dots denote the observations and orange dots denote the smoothed states.

uncover hidden associations between actors and better inform the process of radicalization generally [44].

Finally, as this study focused on a far-right extremist group forum the findings may not be applicable to jihadist groups. Groups such as ISIS and Al-Qaeda appear to use CMCs, particularly social media, in more deliberate and strategic methods than far-right groups [23, 49, 52]. Future research is needed with robust comparative samples of posts from various online communications platforms made by these two movements in order to better compare their practices and organizational characteristics. In turn, we may be able to better document the structural differences in online networks and their role in radicalization.

APPENDIX

Kalman filter equations are standard as follows:

$$x_t^{t-1} = \Phi x_{t-1}^{t-1}$$

$$P_t^{t-1} = \Phi P_{t-1}^{t-1} \Phi' + Q$$

$$K = P_t^{t-1} M' (M P_t^{t-1} M' + R)^{-1}$$

$$x_t^t = x_t^{t-1} + K(y_t - M x_t^{t-1})$$

$$P_t^t = P_t^{t-1} - K M P_t^{t-1}$$

Smoothing Kalman filtering is needed to obtain matrices A , B , and C for the EM Algorithm. To this end, we perform the set of backward recursions $t=n, n-1, \dots, 1$ on the following equations [54].

$$J_{t-1} = P_{t-1}^{t-1} \Phi' (P_t^{t-1})^{-1}$$

$$x_{t-1}^n = x_{t-1}^{t-1} + J_{t-1} (x_t^n - \Phi x_{t-1}^{t-1})$$

$$P_{t-1}^n = P_{t-1}^{t-1} + J_{t-1} (P_t^n - P_t^{t-1}) J_{t-1}'$$

$$P_{t-1,t-2}^n = P_{t-1}^{t-1} J_{t-2}' + J_{t-1} (P_{t,t-1}^n - \Phi P_{t-1}^{t-1}) J_{t-2}'$$

$$P_{n,n-1}^n = (I - KM) \Phi P_{n-1}^n$$

$$A = \sum_{t=1}^n (P_{t-1}^n + x_{t-1}^n x_{t-1}^{n'})$$

$$B = \sum_{t=1}^n (P_{t,t-1}^n + x_t^n x_{t-1}^{n'})$$

$$C = \sum_{t=1}^n (P_t^n + x_t^n x_t^{n'})$$

After building the update matrices A , B and C , an updated operator Φ is obtained from

$$\Phi(r+1) = B A^{-1}$$

along with updated covariance matrices

$$Q(r+1) = n^{-1} (C - B A^{-1} B')$$

$$R(r+1) = n^{-1} \sum_{t=1}^n (y_t - M x_t^n)(y_t - M x_t^n)' + M P_t^n M'$$

$$\Sigma(r+1) = p^{-1} \text{Tr}(P_{0,n}) I$$

and vector

$$x_0(r+1) = x_{0,n}$$

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